STUDY OF THE DISPERSAL OF A SHELL WITH ALLOWANCE FOR FRACTURE AND THE ESCAPE OF DETONATION PRODUCTS BETWEEN FRAGMENTS

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UDC 534.222.2+539.375

Many studies [1-13] have been devoted to theoretical investigation of the dispersal of a shell under the influence of detonation products. Despite this, the effect of the fracture of the shell on its maximum velocity remains unclear. Here, we propose a mathematical model which accounts for fracture of the shell and the escape of detonation products between the fragments. It is shown that, in the case of axial detonation, allowing for fracture leads to a reduction in the maximum velocity of the shell by 20-30% compared to the case of the absence of fracture.

1. We will examine a cylindrical shell with an internal radius a^0, and external radius b^0, and a thickness H. A charge of explosive is located inside the shell, while air is located outside. After the detonation wave reaches the outside surface of the shell, it begins to expand rapidly. A complex shock-wave pattern develops inside the shell and the detonation products (DP). This pattern is tentatively depicted in Fig. 1 by the broken lines. Here, the DP occupy region D_1, the solid shell occupies region D_2, the fractured shell occupies region D_3, and air occupies region D_4. The boundary of the shell, where there is a combination discontinuity (CD), is represented by Γ_1, while Γ_2 represents the DP—air contact discontinuity and Γ_3 represents the shock wave in air. Before fracture, the shell is described by the equations of an ideal elastoplastic medium [14]

$$
\begin{gathered}
\rho_2\frac{\partial u_2}{\partial t}=-\frac{\partial p_2}{\partial r}+\frac{\partial S_1}{\partial r}+(k-1)\frac{(S_1-S_2)}{r},\quad \frac{\partial r}{\partial t}=u_2,\\
\frac{\partial \rho_2}{\partial t}+\rho_2\frac{\partial u_2}{\partial r}+(k-1)\frac{\rho_2 u_2}{r}=0,\\
\frac{\partial \varepsilon_2}{\partial t}-\frac{p_2}{\rho_2^2}\frac{\partial \rho_2}{\partial t}-\left(S_1\frac{\partial u_2}{\partial r}+(k-1)S_2\frac{u_2}{r}\right)/\rho_2=0,\\
p_2=(\gamma_2-1)\rho_2\varepsilon_2+c_0^2(\rho_2-\rho_2^0),\\
\frac{\partial S_1'}{\partial t}=2\mu\left(\frac{\partial u_2}{\partial r}+\frac{1}{3\rho_2}\frac{\partial \rho_2}{\partial t}\right),\\
\frac{\partial S_2'}{\partial t}=2\mu\left(\frac{u_2}{r}+\frac{1}{3\rho_2}\frac{\partial \rho_2}{\partial t}\right),\quad S_1'+S_2'+S_3'=0,\\
S_i=\begin{cases}S_i',\ (S_1')^2+(S_2')^2+(S_3')^2\leqslant\frac{2}{3}Y^2,\\ \sqrt{2/3}\,S_i'Y/\sqrt{(S_1')^2+(S_2')^2+(S_3')^2},\end{cases}\quad \sigma_i=S_i-p,
\end{gathered}
\tag{1.1}
$$

where u_2, ρ_2, p_2, ε_2, S_i, σ_i, c_0, Y, and μ are velocity, density, pressure, specific energy, components of the stress deviator and complete stress tensor, sonic velocity, and the yield points and shear modulus of the shell; k = 1, 2, 3 is the symmetry parameter. The DP and air in regions D_1 and D_4 are described by the equations of an ideal gas, with $\gamma = 3$ for the DP and $\gamma = 1.4$ for air [1]:

$$
\begin{gathered}
\rho_1\frac{\partial u_1}{\partial t}=-\frac{\partial p_1}{\partial r},\quad \frac{\partial r}{\partial t}=u_1,\quad \frac{\partial \rho_1}{\partial t}+\rho_1\frac{\partial u_1}{\partial r}+(k-1)\frac{\rho_1 u_1}{r}=0,\\
\frac{\partial \varepsilon_1}{\partial t}-\frac{p_1}{\rho_1^2}\frac{\partial \rho_1}{\partial t}=0,\quad p_1=(\gamma_1-1)\rho_1\varepsilon_1
\end{gathered}
\tag{1.2}
$$

(u_1, ρ_1, ε_1, and p_1 are the velocity, density, specific energy, and pressure of the gas and DP). Equations (1.1) and (1.2) are valid up to the moment of fracture t^*, which is determined from the well-known Taylor criterion [2-4]. In accordance with this criterion, the shell is

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 33-39, July-August, 1989. Original article submitted July 1, 1987; revision submitted March 15, 1988.

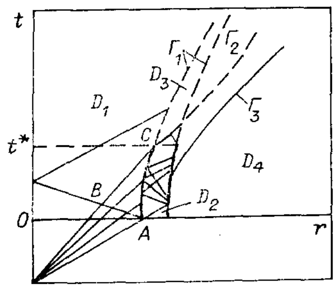

Fig. 1

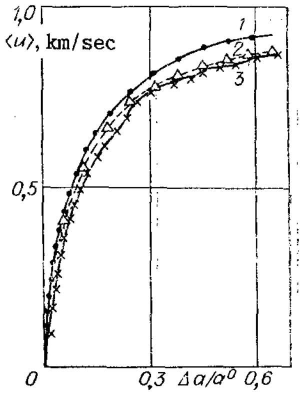

Fig. 2

considered to have fractured if tensile stresses $\sigma_2 > 0$ were acting everywhere within it. Fracture takes place by brittle rupture (radial cracks). Here, the shell fractures into several fragments, the number of which can be found from semi-empirical formulas [10, 11]. For the shell velocities examined in the present study, the number of fragments was on the order of 60. In the region D_3 the fractured shell was modeled by a porous incompressible piston:

$$\begin{aligned}&\frac{d\langle u_2\rangle}{dt}=(p^{+}-p^{-})/(\rho_{22}h),\quad \frac{d\langle r_2\rangle}{dt}=\langle u_2\rangle,\\&p^{\pm}=p_1(\langle r_2\rangle\mp h/2),\ \rho_{22}=\text{const},\ m_2^{*}=(\langle r_2\rangle(t^{*})/\langle r_2\rangle(t))^{(k-1)},\\&m_1^{*}=1-m_2^{*},\end{aligned}\tag{1.3}$$

where $\langle u_2\rangle$, $\langle r_2\rangle$, h, and ρ_{22} are the velocity, coordinate, thickness, and true density of the piston; $p^{\pm}$ is the pressure of the DP to the left and right of the piston; $m_1{}^*$ and $m_2{}^*$ are the minimum porosity and maximum volumetric concentration of particles in the piston. The sudden change in porosity at the inlet and outlet of the porous piston was spread out so that

$$m_2(t,r)=\begin{cases}m_2^{*}(t)\dfrac{r+h'-\langle r_2\rangle}{h'-l}, & \langle r_2\rangle-h'\leqslant r\leqslant\langle r_2\rangle-l,\\ m_2^{*}(t), & \langle r_2\rangle-l<r<\langle r_2\rangle+l,\\ m_2^{*}(t)\dfrac{h'+\langle r_2\rangle-r}{h'-l}, & \langle r_2\rangle+l<r<\langle r_2\rangle+h'\end{cases}\tag{1.4}$$

($\ell = h/4$, $h' - \ell = h/4$ is the width of the region over which the changes in porosity were spread out). In physical terms, this means that each pore can be modeled by a nozzle with a piecewise-linear profile. It should be noted that the width of the spread region does not affect the flow of the DP outside the porous piston if the inequality $h' - \ell \ll \langle r_2\rangle$ is satisfied. The initial conditions for system (1.3) with $t = t^*$ are found from the formulas

$$\langle u_2\rangle=\int\limits_{a(t^*)}^{b(t^*)}u_2\rho_2 r\,dr\Bigg/\int\limits_{a(t^*)}^{b(t^*)}\rho_2 r\,dr,\quad \langle r_2\rangle=(a(t^*)+b(t^*))/2.$$

The equations for DP in a porous piston were obtained from the complete system of equations that describes the flow of a gas in the region of a CD [12]. This system has the form

$$\begin{aligned}&\frac{\partial\rho_1}{\partial t}+\frac{\partial}{\partial r}(\rho_1u_1)+(k-1)\frac{\rho_1u_1}{r}=0,\quad \rho_1=\rho_{11}m_1,\\&\rho_1\frac{\partial u_1}{\partial t}+\frac{\partial}{\partial r}(p_1m_1)=p^{\sigma}\frac{\partial m_1}{\partial r},\quad \frac{\partial r}{\partial t}=u_1,\quad m_1+m_2=1,\\&\frac{\partial\varepsilon_1}{\partial t}+p_1\frac{\partial}{\partial t}\left(\frac{1}{\rho_{11}}\right)=(\langle u_2\rangle-u_1)\left(\frac{p^{\sigma}-p_1}{\rho_1}\right)\frac{\partial m_1}{\partial r},\end{aligned}$$

$$p^{\sigma} = \begin{cases} p_1, \left(\frac{\partial m_1}{\partial r} < 0, \mathrm{M}_{12} < 1, \frac{\partial m_1}{\partial r} > 0, \mathrm{M}_{12} > 1\right), \\ p^{-}, \left(\frac{\partial m_1}{\partial r} > 0, \mathrm{M}_{12} < 1\right), \quad \mathrm{M}_{12} = |u_1 - \langle u_2 \rangle|/c_0, \quad c_0 = \sqrt{\gamma_1 p_1/\rho_{11}}. \end{cases} \quad (1.5)$$

Here, $m_2(t, r)$ is determined from (1.4), while $\langle u_2 \rangle$ is determined from (1.3); M_{12} is the Mach number; ρ_{11} is the true density of the gas. Using system (1.1)-(1.5), we formulate a problem in the region $0 \le t < +\infty$, $0 \le r < +\infty$ with the initial conditions $u_1 = u_1(r)$, $\rho_1 = \rho_1(r)$, $\varepsilon_1 = \varepsilon(r)$, $0 \le r < a^0$, $u_2 = 0$, $\rho_2 = \rho_2{}^0$, $\varepsilon_2 = 0$, $\sigma_i = 0$, $a^0 < r < b^0$, $u_1 = 0$, $\rho_1 = \rho_1{}^0$, $\varepsilon_1 = \varepsilon_1{}^0$, $r > b^0$ and boundary condition $u_1(r = 0) = 0$. The velocities and normal stresses are equal at the contact discontinuities. With the given initial and boundary conditions, system (1.1)-(1.5) is solved numerically by an explicit "cross" scheme [13, 14] with first-order accuracy $O(\tau, \Delta h)$ (τ and Δh are the time and space steps).

2. We will examine two examples of the dispersal of a shell in the instantaneous detonation regime in the absence of air ($\rho_1{}^0 = 0$).

Example 1. Test Calculation of the Dispersal of an Aluminum Shell up to the Moment of Its Fracture. The characteristics of the shell material: $\rho_2{}^0 = 2.7$ g/cm³, $\gamma_2 = 2.18$, $\mu = 0.25 \cdot 10^2$ GPa, Y = 0.3 GPa, shell thickness h = 0.26 cm. The DP, with the parameters $\gamma_1 = 3$, $\rho^0 = 0.68$ g/cm³, $\varepsilon^0 = 4.2$ kJ/g, were located in a cylinder of the radius $a^0 = 0.36$ cm. Here, $\beta = 0.127$, where $\beta = m/M$, m being the mass of the DP and M the mass of the shell. Figure 2 shows the dependence of the velocity of the shell on the relative radius $\langle u \rangle$ ($\Delta a/a^0$). Curves 1 and 2 are described by Eq. (2.1) from [2] without (Y = 0) and with (Y ≠ 0) allowance for strength. Curve 3 corresponds to the numerical calculation.

In our notation,

$$\langle u \rangle = D \sqrt{\frac{\beta}{8}\left(1 - (a^0/a)^4\right) - \frac{2Y}{\rho^0 D^2} \ln (a/a^0)}, \quad D = \sqrt{8(\gamma_1^2 - 1)\varepsilon^0} \quad (2.1)$$

(D is the velocity of the detonation wave). It follows from Fig. 2 that the calculated velocity of the shell is adequately described by the analytical relation $\langle u \rangle$ (a/a^0) obtained for an incompressible ideally plastic shell.

Example 2. Dispersal of a Cylindrical Copper Shell under the Influence of DP Formed after the Detonation of Octogen. This problem was studied experimentally in [9]. The shell was a long thin tube with $a^0 = 1.27$ cm, $b^0 = 1.53$ cm, and the length L = 30 cm. Detonation was initiated from one end, while the rate of expansion of the shell was recorded in the midsection 15 cm from the end of the tube. In this case, we could ignore the effect of the escape of DP through the end surface of the tube on the rate of its expansion in the midsection. This, in turn, allowed us to compare experimental and theoretical dependences of the velocity of dispersal of the shell on time. Figure 3 shows the relation $\langle u \rangle(t)$ for the case when the DP were formed by octogen with the parameters $\gamma_1 = 3$, $\rho^0 = 1.89$ g/cm³, and $\varepsilon^0 =$ 5 kJ/g and the characteristics of the copper shell were $\rho_2{}^0 = 8.93$ g/cm³, $c_0 = 3.93$ km/sec, $\gamma_2 = 2.69$, $\mu = 0.39 \cdot 10^2$ GPa, Y = 1.8 GPa, $\beta = 0.5$. The curve represents our calculation, while the points represent the experimental results [9]. The shell fractured at the moment $t^* = 6$ µsec. This was followed by the intensive outflow of DP in a regime that can be characterized by choking of the flow in the channels of the porous piston [12, 15]. In the minimum cross section (minimum m_1), $M_{12} = 1$. To the left of this section, $M_{12} < 1$, while to the right $M_{12} \gg 1$. Here $u_1 - \langle u_2 \rangle > 0$. Allowing for fracture and the escape of DP leads to a small (5-6%) reduction in the maximum velocity of the shell compared to the case without fracture. We performed six calculations for different DP parameters corresponding to [9]. We obtained good agreement with the experimental time dependence of shell velocity $\langle u \rangle(t)$.

3. Let us examine the dispersal of a copper shell in a regime whereby detonation is initiated on the axis. The characteristics of the shell material and the geometric parameters of the problem are the same as in example 2. The DP were formed by the detonation of octogen with additives: $\rho^0 = 1.862$ g/cm³, D = 8.82 km/sec [1]. The initial conditions for the DP were determined by numerical integration of system of ordinary differential equations (5.151)-(5.152) from [1], which describes the corresponding similarity solution. In the present formulation, we studied the effect of fracture of the shell on its velocity and on the degree of dissipation of the energy of the DP by the shell. The calculations were performed with several values of β. The thickness of the shell was also varied. For each

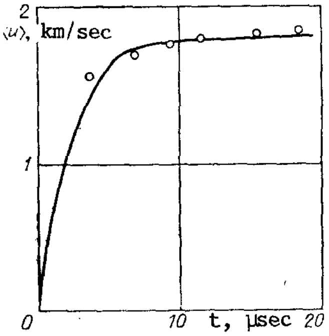

Fig. 3

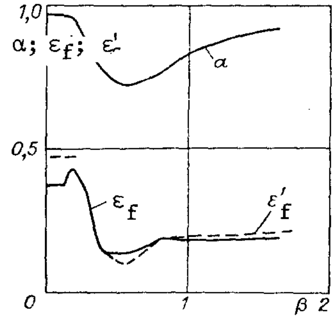

Fig. 4

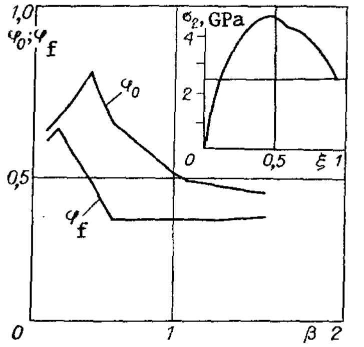

Fig. 5

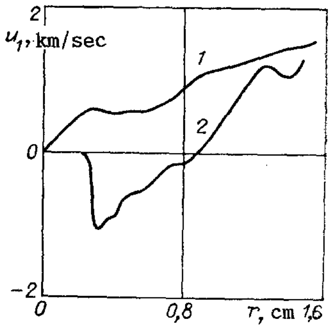

Fig. 6

β, we calculated the mean-mass velocity of the shell with v_f and without v_0 fracture at the moment t = 20 μsec. The shell has had sufficient time to reach its maximum velocity by this point. We will determine the ratio $\alpha = v_f/v_0$ and construct α as a function of β (Fig. 4). It turns out that the value of α decreases from 0.97 to 0.73 in the interval $0.3 < \beta < 1$. Thus, fracture of the shell and the escape of DP between the fragments leads on this interval to a substantial reduction in shell velocity and the degree of energy dissipation $\varphi = Mu^2/(2m\varepsilon^0) = 8u^2/(\beta D^2)$.

Figure 5 shows the ratio $\varphi(\beta)$:$\varphi_0(\beta)$ for a shell without fracture and $\varphi_f(\beta)$ for a shell with fracture. It is evident that the peak of the curve $\varphi_0(\beta)$ at the point $\beta = 0.4$ completely disppears in the case of a fractured shell. The large trough on the curve $\alpha(\beta)$ is due to the fact that an increase in β from 0.2 to 0.4 is accompanied by a sharp decrease (by a factor greater than two) in the critical strain ε_f. This quantity is found from the formula $\varepsilon_f = (a^+ - a^0)/a^0$ (a^+ is the internal radius of the shell at which it fractures). Along with $\varepsilon_f(\beta)$, there is also a large reduction in the time to fracture. Thus, at $\beta < 0.2$, the characteristic time to fracture $t^* \simeq$ 7-8 μsec, while $t^* \simeq$ 1.3-2 μsec on the interval $0.3 < \beta < 1.66$. In the region $\beta > 0.6$, there is an increase in $\alpha(\beta)$. This increase is connected with the fact that there is sufficient time for the shell to reach a high velocity before it fractures. In this case, $\varepsilon_f(\beta)$ remains nearly constant. The reduction in $\varepsilon_f(\beta)$ at $0.3 < \beta < 0.4$ is connected with the occurrence of wave processes in the DP. At $\beta < 0.2$, there is sufficient time for the shock wave reflected from the shell to be reflected from the axis and catch up with the shell before it fractures. Thus, the entire region occupied by the DP participates in the shell's acceleration. At $\beta > 0.4$, the shell fractures before the shock wave reflected from the center catches up with the shell. In this case, only that part of the DP adjacent to the surface of the shell participates in its acceleration. After the shell fractures due to the escape of the DP and the pressure drop, its acceleration nearly ceases. In the case of the acceleration of a solid shell, the reflected shock wave catches up with the shell and produces an additional acceleration.

Figure 6 shows the distributions of velocity $u_1(r)$ in the DP for $\beta = 0.57$ at the moment of fracture (curve 1 corresponds to instantaneous detonation, while curve 2 corresponds to axial detonation). In the case of instantaneous detonation, the entire DP region participates in the acceleration ($u_1 > 0$). In the case of axial detonation, only that part of the DP within the layer 0.9 cm < r < 1.44 cm participates in acceleration. In the layer 0.25 cm <

TABLE 1

β	$\langle u \rangle$, cm/μsec		β	$\langle u \rangle$, cm/μsec	
	theory	calculation		theory	calculation
0,1	$8{,}2\cdot10^{-2}$	$7{,}35\cdot10^{-2}$	0,8	0,157	0,164
0,13	$9{,}4\cdot10^{-2}$	$8{,}4\cdot10^{-2}$	1	0,179	0,188
0,18	0,11	$1{,}04\cdot10^{-1}$	1,66	0,24	0,236
0,57	0,1	0,133	4,6	0,37	0,3

$r < 0.9$ cm, the DP move toward the axis of the charge ($u_1 < 0$), while the DP are at rest ($u_1 = 0$) in the layer $0 < r < 0.25$ cm. It follows from analysis of the data that ε_f and α depend on the form of the pressure pulse in the DP and the characteristics of the shell material. Thus, $\varepsilon_f = 0.37$ and $\alpha = 0.89$ with instantaneous detonation, while $\varepsilon_f = 0.14$ and $\alpha = 0.72$ with axial detonation. In both calculations, $\beta = 0.57$. It should be noted that the wave patterns in both the DP (Fig. 6) and in the shell differ significantly in these cases. In the case of instantaneous detonation, the relation $\sigma_2(r)$ is approximated well by a linear function which reaches a maximum on the outside surface of the shell [2]. For axial detonation, this relation deviates greatly from linearity.

Figure 5 shows the relation $\sigma_2(\xi)$ at the moment $t = t^*$ ($\beta = 0.57$) [$\xi = (r - a)/h$]. The maximum of $\sigma_2(\xi)$ is reached inside the shell and is more than twice as great as the yield point Y and the fracture stress σ_f [1]. In the model proposed here, we ignored cleavage in the shell under the influence of radial tensile stresses [1]. This is connected with the fact that the fracture of a cylindrical shell is affected mainly by wave processes in the DP and the yield point of the shell material. Thus, the effect of cleavage on the fracture of the shell will probably be small.

4. Let us give a theoretical estimate of the critical strain and velocity of a shell in the case of axial deformation for arbitrary β. With small β, the velocity of the shell can be calculated from (2.1) without allowance for strength ($Y = 0$). The value of ε_f can be calculated from the equation [2]

$$a^+/a^0 = (p_0/Y)^{1/(2\gamma_1)}, \quad p_0 = p_e/2, \quad \varepsilon_f = (a^+ - a^0)/a^0. \tag{4.1}$$

Equations (2.1) and (4.1) were obtained in the instantaneous detonation approximation. Thus, as follows from the calculations, they are valid for $\beta < 0.2$. At $\beta \geq 0.4$, wave processes in the DP become important. To describe these processes, we use the analytical solution for the acceleration of an incompressible piston by a detonation wave in the plane case [1]. It is clear that the solution will be approximate for a cylindrical shell. Nonetheless, the approximation turns out to be satisfactory in the given case. This has to do partly with the fact that the distribution of the parameters of the DP behind the detonation wave front is similar in the plane and cylindrical cases. Also, it follows from the calculations that the relative expansion of the shell before fracture is small ($\varepsilon_f \lesssim 0.2$), so the corresponding change in the Riemann invariant is also small. In accordance with [1], for the cylindrical case and $\gamma_1 = 3$ we have $d(u_1 + c) = -(u_1 c/r)dt$. Proceeding on the basis of this, we obtain $|\Delta(u_1 + c)/(u_1 + c)| \lesssim \langle u \rangle \Delta t/r \simeq \varepsilon_f \ll 1$ in the region ABC along the characteristic $dr = (u + c)dt$. Before the solution from [1] is used, it must be noted that the detonation wave is being reflected from a copper shell, not a rigid wall, and the pressure in the reflected wave increases by a factor of 1.56. Allowing for this fact and the cylindrical symmetry of the problem, we rewrite the formulas from [1] in the form

$$\begin{gathered} c = a^0\theta/t, \ \theta = (1 + 2\eta(1 - a^0/Dt))^{-1/2}, \\ a = Dt(1 - (1 - \theta)/\eta\theta), \\ \langle u \rangle = D(1 - (1 - \theta)/\eta\theta) - a^0\theta/t, \ \eta = k\beta/2, \\ p = kp_e(c/D)^3, \ p_e = \rho_0 D^2/(\gamma_1 + 1), \end{gathered} \tag{4.2}$$

where c and p are the sonic velocity and the pressure on the surface of the shell; $\langle u \rangle$ and are the velocity and coordinate of the shell; k is the coefficient of reflection of the shock wave. The amount of fracture of the shell is determined from the Taylor condition $p = Y$. With the use of (4.2), it follows from this that

$$kp_e(a^0/Dt')^3/(1 + 2\eta(1 - a^0/Dt'))^{3/2} = Y \quad (t' = t^* + a^0/D).$$

Solving this equation for t', we obtain

$$t' = \frac{a^0}{D}\Big/\left(\sqrt{\eta^2\zeta^{4/3} + (1 + 2\eta)\zeta^{2/3}} - \eta\zeta^{2/3}\right) \quad (\zeta = Y/kp_e). \tag{4.3}$$

We used Eqs. (4.1)-(4.3) for several values of β to calculate the critical strains ε_f' and the velocity of the shell ⟨u⟩. For shells with $\beta < 0.2$, ε_f' was calculated from Eq. (4.1). For $\beta \geq 0.4$, it was calculated from (4.2) and (4.3). In the latter case, we assumed that $k = 1.56$, $a^0 = 1.3$ cm, $D = 0.882$ cm/μsec, $p_e = 0.362 \cdot 10^2$ GPa, $Y = 1.8$ GPa, $\eta = 0.78\beta$, and $\zeta = 0.032$. The relation $\varepsilon_f'(\beta)$, shown by the dashed line in Fig. 4, agrees well with $\varepsilon_f(\beta)$ obtained from the numerical calculations. Table 1 shows the velocity of the shell ⟨u⟩ at the moment of fracture for several values of β. The second column of the table gives the values found from (2.1) and (4.1) with $\beta \leq 0.18$ and from (4.2) and (4.3) with $\beta \geq 0.57$. The third column shows values obtained from the numerical calculations. It is evident that with fixed β the velocities are fairly close to one another. The shell does not reach more than 10% of the maximum velocity after fracture due to the escape of the detonation products. This makes it possible to use Eqs. (2.1) and (4.1)-(4.3) to evaluate the velocity of the shell in engineering calculations. Here, we chose the Taylor fracture criterion, which is valid for thick shells [3, 4]. In the case of thin shells, it is necessary to use the energy criterion in [10]. This leads to some change in the velocity of the shell and in fracture strain. In particular, the scale effect is manifest as a result of the use of this criterion.

In conclusion, we note that the observed phenomenon of a substantial reduction in shell velocity due to fracture will also evidently hold for thin shells, since it is due to wave processes occurring in the detonation products. However, this issue requires additional study.

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